

IV

HARMONY

1. THE PROBLEM OF CHORDAL SEQUENCES

WHEN two chords of the diatonic scale, no matter how pleasing in themselves, follow one another without due regard to the progression of the various parts, the effect is unsatisfactory, and the sequence is unsuitable for musical purposes. For instance, if the dominant four-part chord in fundamental position G-g-b-d, be followed by the adjacent subdominant chord F-f-a-c, the sequence obtained is disagreeable notwithstanding the fact that both of the chords are highly satisfactory. Thus the varying æsthetic quality of chordal sequences depends not only upon the æsthetic factors involved in the constituent chords but on other factors involved in the method of transition.

The classification of such chordal sequences presents an important æsthetic problem, fundamental for harmony, which is much more complicated than that of the single chord. It is our aim to deal with this problem of chordal sequences from the point of view of æsthetic measure.

For the sake of definiteness we shall confine attention to the important case of diatonic harmony in four distinct parts. The key will be taken as that of C major.

We shall employ an arrow to indicate the direction of the progression; thus Ia \rightarrow Vb would indicate a sequence in which the passage was from the tonic chord in fundamental position to the dominant chord in its first inversion. Use

will sometimes be made of the double arrow \longleftrightarrow , as for instance $I \longleftrightarrow V$, to indicate that the sequence may be taken in both orders.

2. THE METHOD OF ATTACK

In order to obtain an æsthetic formula for chordal sequences, we need first to obtain a suitable definition of the complexity C . Now it has been pointed out that the complexity C of all single chords must be regarded as the same. Evidently the complexity of all sequences of two chords must also be taken to be the same. For definiteness we adopt the value 1 for C . With this simplification the basic formula, $M = O/C$, reduces to the simpler formula: $M = O$. In other words the problem is reduced to the determination of the elements of order O involved in such sequences.

We propose to attribute the total æsthetic effect to the individual effects of the chords together with the effect of the transition from the first chord to the second; thus, if m_1 and m_2 denote the æsthetic measures of the first and second chords respectively, as determined by the preceding theory, and if t denotes the similar æsthetic measure of the transition, we shall regard the æsthetic measure of the sequence M as given by the sum of these three components:

$$M = m_1 + t + m_2$$

The fundamental problem before us is that of specifying the transition measure t .

3. FURTHER LIMITATION OF THE PROBLEM

It would be desirable to present a theory applicable to all chordal sequences whatsoever, or at least to all sequences in which the individual chords are regular.

296 Mathematical Theory of Aesthetics

Nevertheless it is proposed here to consider only those "regular chordal sequences" in which (1) the leaps in the individual voices are not excessive, and (2) the similarity in function of the voices is not too great.

Of course it is in no way suggested that a slightly extended theory would not suffice to deal with all chordal sequences. Such a theory would include two further negative elements of order, corresponding to excessive leaps and to excessive similarity.

The arbitrary and fluidic character of the specific limitations which we state below must be strongly emphasized. In general, other things being equal, it is desirable that the leaps in the voices are not large and that the voices perform dissimilar functions. But, the moment any specified limitation of this kind interferes with free musical expression, it is to be ignored.

Nevertheless the great body of classical music does observe such limitations.

4. THE LIMITATION OF LEAPS

The urgency for some limitation as to leaps is obvious. If the leaps are excessive, not only are the voices likely to fall outside of their natural compasses, but an unusual effort is required to make such a leap, and this affects the hearer unpleasantly. The step of a second or the leap of a third up or down is more easily made in general than a leap of a larger interval. Scarcely ever is an interval of as much as an octave used, and this ordinarily occurs in the bass. The rules to be stated presently are such as to take account of these and similar facts, and thus to insure reasonable ease of execution in singing the parts.

Inasmuch as the bass part frequently occurs as the root of a primary chord, and the progressions between roots of

primary chords are mainly perfect fourths or fifths, up or down, the bass voice becomes used to these leaps. In fact, such a "harmonic step" in the bass voice is made almost as easily as a "melodic step" in the upper voices. The leap of an octave is also made frequently in the bass part in order to avoid a stationary note.

In the upper parts a leap of as much as an octave usually occurs only when the second chord is another position of the first. In this event there is greater ease of execution of the leap than there is when the chord changes. Only one leap of a fourth or more is agreeable in the upper parts when the chord changes, however, and if the bass leaps by as much as a sixth, no leap in the upper parts should be more than a third.

In general, no part should leap by a dissonant interval. An obvious reason for this feeling is that the dissonant interval is unpleasantly suggested; a further and perhaps more important reason is that the consonant intervals are the ones of natural harmonic origin and are therefore more easily sung.

There are, however, two exceptions. In the first place the interval of the diminished fifth (B up to F, or F down to B), which is only slightly dissonant and which is a pleasing interval, is tolerated. However, the equivalent augmented fourth (B down to F, or F up to B) is not allowed. The distinction felt may be explained as follows: both notes involved are harmonically related to one another because they are essentially overtones of the dominant; however, as an overtone of the dominant, the leading note is lower in position than the subdominant, and hence the interval is only tolerated when the notes are taken in this natural order.

In the second place the leap of a minor seventh, from the

298 Mathematical Theory of Aesthetics

dominant up to the subdominant or *vice versa* is permitted for a similar reason. This dissonant interval lies in the constantly used dominant 7th chord. However, this leap only occurs in the extreme parts, since in the middle parts it leads to forbidden crossing of the voices.

The reason why such crossing or overlapping of the parts is avoided is that any voice has a tendency to stop at a part just taken by another voice rather than to proceed beyond it. The effort required to resist this natural tendency is distinctly noticeable.

Such are the usual limitations governing the leaps in the parts, and we shall embody them in the following simple rules:

No part is to leap across a note just sounded in another part. When the chord does not change, there is no further restriction in the motion of the parts.

Otherwise, at most one leap of a fourth or more is allowed in the upper parts, and this is to be less than an octave. No leap of more than a twelfth (octave plus fifth) is allowed in the bass. If the leap in the bass is as much as a sixth, no leap in the upper parts exceeds a third.

Leaps are to be by a consonant interval, with the possible exception of a leap of a diminished fifth, or of a minor seventh to or from the dominant.

One other requirement may be mentioned although, strictly speaking, it does not apply to a sequence of only two chords: if a part leaps by as much as a sixth in one direction, it is desirable for the part to proceed in the opposite direction of motion; obviously this reversal of direction relieves the unusual vocal effort involved.

Such arbitrary rules are often successfully broken. But, inasmuch as we are confining attention to musical problems *with a definite allowance of means*, it is permissible to

impose requirements which are generally observed in the large body of classical music.

5. LIMITATION OF SIMILARITY OF FUNCTION

The basic reason for some limitation of the similarity of function in two voices lies in the requirement of a *homogeneous* use of the voices in musical composition: either the voices should be similarly related throughout, as by successive octaves, or not so related anywhere in a musical composition. Thus distaste for certain types of similarity has taken definite form.

The similarity between two voices varies with the degree of harmonic relationship between the notes involved and is enhanced by "similar motion," when both voices rise or both fall. On the other hand, any similarity is much less marked in the case of contrary motion, and is felt to be less objectionable between the strong primary chords and in the inconspicuous middle voices. Such are some of the general considerations which have operated to determine the usual practice.

The entirely explicit rules of Prout (P., pp. 25–33) are not formulated so that any associative basis for their curious structure is obvious. A nearly equivalent formulation, which aims to display the basic associative structure is as follows:

Consecutive octaves and consecutive perfect fifths are forbidden in the case of similar motion (when there is almost complete identity of function). In the case of consecutive octaves they are allowed¹ by contrary motion between the primary chords with doubled primary notes (the best chords of all). In the case of consecutive perfect fifths it is required further that the extreme parts be not involved (because of

¹ Providing that the sequence is satisfactory *otherwise*.

300 Mathematical Theory of Aesthetics

the greater conspicuousness of the perfect fifth in these parts).

Consecutive perfect fourths are only forbidden by similar motion when one part is in the (prominent) bass part (for the third above the bass and *not* the fourth is desired).

Hidden octaves are forbidden when the voices move from a dissonant seventh or ninth to the octave (since the prominent dissonance is not felt to be properly resolved). Otherwise hidden octaves and hidden perfect fifths are forbidden only when in the (conspicuous) extreme parts. However, in case the second chord is a primary chord with doubled primary note (one of the best chords) they are tolerable as follows: in the case of hidden octaves, when both chords are primary chords in fundamental position and the upper part moves by step only (thus diminishing the similarity of function since the bass leaps); and when the second primary chord is not in fundamental position (so that the octave in question does not involve the conspicuous root and bass of the chord); in the case of hidden perfect fifths, when the first chord is also primary and the upper part moves by step (thus diminishing the similarity of function since the bass leaps); and when both chords form a dominant sequence (and so lie practically in the same chord).¹

We shall not attempt to show that these general implicit limitations are essentially coextensive with Prout's entirely explicit rules.

6. THE LAW OF RESOLUTION

It is necessary to recall one further important musical phenomenon before proceeding to enumerate the elements of order involved in chordal sequences, namely that of resolution of dissonance.

¹ The only case of this sort is $iia \rightarrow Va$ with the third of iia in the soprano.

Music involving only consonant chords is lacking in interest to the modern ear. In fact dissonance, followed appropriately by consonance, is a most important method of obtaining contrast. In classical music the extent to which such resolution of dissonance may be effected has been rather clearly defined.

Let us consider a few very simple instances.

The notes of the dissonant interval B-f are seen each to be only a semitone distant from these of the consonant interval C-e. If the interval B-f be followed by the consonant interval C-e, the ear is satisfied and B-f is said to be "resolved" on c-e. Similarly, when the dominant 7th chord G-B-f goes to G-c-e, with B-f moving to c-e as before, while G is stationary, V7 is resolved on I. Or if we take the interval of the minor seventh G-f of V7 and follow it by the interval G-e of I the dissonance is resolved on I by allowing the dissonant note F to move to the nearest note of the tonic chord, E, while the note G is held.

Evidently in these examples, the dissonant notes of the chord move to the *nearest* positions of a consonant chord.

On the other hand, if we take the interval of the ninth C-d and allow it to go to the octave C-c, the effect is not satisfying. Here the dissonant ninth moves to a dissonant note. If we allow the note d to rise to e the effect is improved but still unsatisfactory because of the harshly dissonant quality of the interval of the ninth.

By comparison of various cases of resolution, these are found to obey a general "law," which may be formulated in the following way.

A dissonant chord goes to a consonant chord as follows: (1) a dissonant note of the first chord not found in the second chord goes to a nearest note of the second chord, which must not be a (dissonant) note of the first chord;

302 Mathematical Theory of Aesthetics

(2) if the particular note thus designated has been taken by the bass part, the note is free to move a step in the opposite direction; (3) the second chord must be the tonic, subdominant, or submediant chord *via* in case the first chord is $V7$ or its derivative $viib^1$; in all other cases it must be the dominant, tonic, or submediant chord *via*.

In connection with the requirement (2) we observe that the chord $V7$ and its derivative $viib$ are too closely related to the dominant chord V to resolve satisfactorily upon V ; furthermore, resolution upon the submediant *via* is permitted because this chord is felt to be a kind of substitute for the tonic chord; in fact *via* has two notes in common with the tonic chord and is indeed the fundamental position of the tonic chord in the related minor key.

The manner of handling dissonance is not always to pass directly from a dissonant chord to a consonant chord. In fact sequences of dissonant dominant chords and of their derivatives are often employed in the following order:

$$V13 \rightarrow V11 \rightarrow V9 \rightarrow V, V7, Viib \rightarrow I \text{ or } via.$$

Here some of the chords may be omitted, or the ending may be of the form $V7 \rightarrow IV \rightarrow I$. This or any other logical sequence of dissonant dominant chords may be employed in a musical composition. In such a case the resolution is of course effected at the moment of passing to a consonant chord by the method prescribed in the rule.

7. THE ELEMENT OF RESOLUTION: $R = 4, - 4$

We are now able to formulate briefly the various elements of order which (in our opinion) determine the æsthetic measure of transition, t .

Let us begin with the element of resolution, R , which is

¹ We do not consider the chord $viia$ which is not passable.

present when a dissonant chord proceeds to a consonant chord in accordance with the law of resolution of the preceding section.

We shall take R to have the index 4 if resolution is properly effected; if it is not so effected in a sequence of two such chords we shall take R to be -4 , since the effect is very unsatisfactory. In any chordal sequence not of this type R is 0 of course.

The index 4 seems to be that which rates this element suitably in connection with the indices used in the æsthetic measure of the single chord. This means that the element R in the transitional effect is rated as about equally important with that of the effect of a single good chord ($m = 4$ or 5).

8. THE CADENTIAL ELEMENT: $Cl = 4, -2$

“Cadence” may be regarded as a very mild form of resolution. For example, if we pass from the dissonant dominant seventh to the tonic, $V7 \rightarrow I$, there is definite resolution. If we pass from the closely related consonant dominant to the tonic, $V \rightarrow I$, a similar cadential effect is felt. The two fundamental forms of cadence are the above so-called “authentic cadence” $V \rightarrow I$ and the less frequent “plagal cadence” $IV \rightarrow I$. The authentic form is usually found at the end of a musical composition; the plagal form is also employed, but much less frequently, as in the familiar *Amen*.

By a “cadential sequence” we shall mean, however, not only sequence of these types $V \rightarrow I$, $IV \rightarrow I$ but also certain obviously analogous sequences, which we proceed to enumerate.

In the first place, the submediant in fundamental position, *via*, may be regarded as a kind of substitute for the tonic chord (see the preceding section). Thus the sequence $V \rightarrow \text{via}$ is the frequently used “interrupted cadence” and

304 Mathematical Theory of Aesthetics

has a definite cadential effect. Similarly the chord on the leading note $viib$ functions in many instances as $V7$ (V) so that the sequences $viib \rightarrow I$, via are felt to be cadential. In the case $viib \rightarrow via$, however, when neither chord is primary, the leading note of $viib$ must go to the tonic of via for a satisfactory cadential effect to be produced.

The cases so far enumerated have the authentic cadence $V \rightarrow I$ for their prototype. There is one more of this general character, namely the sequence from mediant to submediant, $iii \rightarrow vi$, for this functions as the authentic cadence in the related minor key.

In the case of the less important plagal cadence, the only analogous form felt to have cadential quality is that of the related minor key, $ii \rightarrow vi$.

We shall regard all of these types of sequences as cadential, and the corresponding cadential element of order Cl will be given the index 4, the same as the index for the element of resolution R .

Moreover we shall regard the same types of sequences, *taken in the reverse order*, as cadential, so that Cl is 4 in these cases also.

The question may be asked as to why the inverted order is allowed here, although these are not ordinarily considered to be cadential. This may be explained in part as follows. The plagal cadence is equivalent to an inverted form of the authentic cadence as far as the intervals involved are concerned; this reversibility extends by association to the other forms.

It remains to refer to two other cases when an unpleasant or *false* cadential effect is produced. In the first case the leading tone goes to the tonic ($7 \rightarrow 1$) in an extreme part, as usually happens in the fundamental direct form of authentic cadence $V \rightarrow I$, and yet there is no cadence of the above

named types; under these circumstances there is a feeling of disappointed expectation. In the second case, IV in fundamental position goes to the related tonic minor via in fundamental position; thereby a plagal cadence $IV \rightarrow I$ is suggested, but the *two* common notes of IV and vi negate the cadential effect so strongly that the effect is again one of disappointed expectation.

Thus the final types of positive cadential sequences are the following:

V, viib \longleftrightarrow I, via ($7 \longleftrightarrow 1$ in viib \longleftrightarrow via), iii \longleftrightarrow vi,
IV \longleftrightarrow I, ii \longleftrightarrow vi.

For these, *Cl* is 4.

The two types of false or negative cadential sequences are the following:

$7 \rightarrow 1$ in extreme part but without cadence,
IVa \rightarrow via,

For these, *Cl* is -2 .

In any other case *Cl* is taken to be 0.

9. THE ELEMENT OF DOMINANT SEQUENCE: $D = 4$

In the classification of dominant chords V, V7, V9, V11, V13, it has been seen in the preceding chapter that certain forms of these are to be regarded as passable. When a sequence of two of these are heard, they are pleasantly united by a common dominant quality whose importance is obvious.

Moreover certain consonant chords besides V may be imbued with dominant quality. The principal consonant chords of this kind are: viib, which may be regarded as a derivative of V7; ii, as a derivative of V9; and iii as a derivative of V13.

It is to be noted that the first inversion of iii has the

306 Mathematical Theory of Aesthetics

dominant in the bass and in its only passable form has three of its four notes in the dominant chord.

However in the sequence V7, viib \rightarrow V there is little or no sense of dominant *sequence* since viib appears naturally as part of the slightly dissonant V7 which includes V and calls for resolution on the tonic chord. On the other hand $V \rightarrow V7$, viib forms dominant sequences since V7, viib extend beyond V.

Moreover, whenever the chord viib comes first, its weak dominant character is not felt when it is followed by ii, iii. Likewise a sequence $ii \longleftrightarrow iii$ is not felt to have dominant quality since ii is not strongly dominant and iii is scarcely recognized as dominant. Under such circumstances the element of dominant sequence 0 is taken to be absent ($D = 0$).

In case ii comes first in $ii \rightarrow V$, viib, the submediant 6 must proceed to the adjacent 5 or 7, as in the resolution of V9. Likewise if iii comes first in $iii \rightarrow V$, viib, the mediant 3 must proceed to the nearest available note 4 or 2; and in case iii is in its weak dominant position $iiia$, the note 3 must be in the soprano (as in the case of V13) and must proceed to the nearest note as in a resolution of V13. If these additional conditions are not satisfied D is taken to be 0.

Hence, subject to the further conditions just stated, we regard the element D of dominant sequence as present in any sequence of two chords ii, iii, viib, V, V7, V9, V11, V13, excepting $viib \rightarrow ii, iii, V$ and $ii \longleftrightarrow iii$.

Under these circumstances D is taken as 4, and otherwise as 0.

10. THE ELEMENTS $SF = 4$ AND $RF = 4$

In passing from one three-part consonant chord in closest possible position to another, there are certain special cases

in which the transition is unusually smooth. These are those in which certain undesirable contingencies are avoided, namely (1) stationary bass or soprano, (2) forbidden crossing of the parts, (3) forbidden consecutive perfect fifths, (4) second inversions, which are only passable for the primary chords.

An examination of the various possibilities is easily made. If, for example, we designate root, third and fifth of the first chord as 1, 3, 5 we find the only following five general possibilities:

$$3-5-1 \rightarrow 4-6-2, 5-7-3, 2-4-7, 1-3-6; 1-3-5 \rightarrow 2-4-7$$

Hence such "close harmony" arises in the case of successive first inversions in which the root falls or rises by one or two steps (*FF*), and also when the first chord in root position is followed by a first inversion with root falling one step (*RF*). In the first case we say that the element *FF* is present, and in the second case that the element *RF* is present. Both of these elements are given an index 4.

It will be found that the usual rules are consistent with the assignment of definite æsthetic value to these special types *FF* and *RF* of root progressions.

The smoothness of a chordal progression *FF* may be verified by playing an arbitrary set of first inversions in which the root steps or leaps two steps as one desires. The type *RF* is equally smooth but does not admit of continuation in such a progression because there is no means of returning to fundamental position, once it has been left.

11. THE ELEMENT OF PROGRESSION: $P = 2, 0 = 2$

If, in passing from the first chord of a sequence to the second, it is possible to "borrow" a note of the first chord as a consonant part of the second, the transition from one to the other is felt not to be unnatural.

308 Mathematical Theory of Aesthetics

If two or more notes can be so borrowed, there is felt to be a sense of positive harmonic progression in passing from the first chord to the second; for the anticipation provoked by the first chord is then fulfilled in a suitable manner.

Since V is so closely associated with viib, through V7, it is also permitted to borrow the dominant note for use in viib.

Moreover an effect of melodic progression is obtained when the two chords have no notes in common provided that each note of the first chord proceeds to a nearest note of the second chord, except in so far as the bass or an adjacent note or an octave of this note has proceeded to such a nearest note, in which case the note is free to move a step in the opposite direction. Furthermore the tonic may leap a third in this direction, and a note freed by its octave may move freely in the same direction.

In all of these cases P is taken to be 2, provided the first chord does not resolve on the second.

Otherwise, if there is resolution, or if the sequence is dominant, or if one and only one note can be borrowed, P is taken to be 0.

In every other case the two chords appear as unrelated and P is taken to be -2 .

In illustration of progression value P , we may consider the following sequence of chords $I \rightarrow IV \rightarrow vii \rightarrow iii \rightarrow vi \rightarrow ii \rightarrow V \rightarrow I$.

The musical notation illustrates a sequence of chords: I, IV, vii, iii, vi, ii, V, and I. Above the staff, the progression values P are indicated for each transition: 2, 2, 0, 2, 2, 2, 2. Below the staff, the resolution values are indicated: 0, 0, 0, 0, 0, 0, 0. Arrows labeled P point to the first and last chords.

This sequence conveys a sense of harmonic progression, whereas in the reverse order, all sense of progression disappears.¹

This evaluation of the progression value P seems to be satisfactory.

The simple positive elements R , Cl , D , FF , RF and P are all of unquestionable æsthetic importance. These are the only positive elements which we shall take account of in our theory. The remaining elements are all negative in their æsthetic effect.

12. THE NEGATIVE ELEMENT: $FR = 4$

If the first chord is a first inversion and the second chord is in fundamental position while the root falls one step, the effect is in general unsatisfactory.

In fact this is the only type of root progression which, if executed in "close harmony," by three voices, requires two forbidden crossings of the voices and gives rise to chords without common note of connection.

For such a sequence we take the element FR to be 4; in any other case we take FR to be 0.

This element is in part recognized by Prout in his rule *VI* of root progression (p. 30): "As a general rule, whenever the root falls a second, *the second of the two chords should be in its first inversion.*" In other words, the progression is to be of the type FF or RF , and not of the type FR or RR .² This statement, clearly in agreement with our statements above: for the elements FF and RF have been taken as to be positive, while FR has been taken to be negative.

¹ See D. Tweedy, *Manual of Harmonic Technique*, Boston (1928), p. 106.

² That is, a sequence of two fundamental positions with root falling a second.

310 Mathematical Theory of Aesthetics

13. THE NEGATIVE ELEMENT OF THE MEDIANT: $M=2, 4$

In the hierarchy of the fundamental triadic chords, the mediant chord is generally felt to occupy the least favorable position. For, among these chords, the dominant and subdominant chords, being of the same constitution as the central tonic chord are felt to be "primary"; furthermore the leading tone chord and the supertonic chord are dominant enough in quality as to be felt to be related to the primary dominant chord; and of the two remaining chords (the mediant and submediant), the submediant fulfills the important function of substitute for the tonic chord itself. Thus the mediant chord, of doubtful dominant quality in its fundamental position and having no special function, is felt to be of dubious value.

For some such reason the presence of the mediant chord in a chordal sequence produces a wellknown unfavorable effect.

On account of this fact the negative element of the mediant, Mt , with index 2 will be generally attached to any chordal sequence in which the mediant chord is present; in certain exceptional cases, specified below, the index 4 will be assigned.

Furthermore, in case *iii* is not present but the characteristic first and third of *vi* and *iii* appear in the first and second chords respectively while the bass leaps either from 6 or to 3, or does both, all as in a characteristic cadential passage from *vi* to *iii*, we also take Mt to have the index 2.

The reason for this is obvious: under these circumstances, the mediant chord is strongly suggested even though it is not actually present.

However, if *iii* is present and at the same time the preceding conditions are met, we take Mt to have the index 4, since the chord *iii* is then still more strongly suggested.

Finally if there is a passage from the mediant chord in its fundamental position to the central tonic chord in fundamental position, there is a strong effect of disjunction so that Mt is also assigned the index 4 in this case. The inverse passage $Ia \rightarrow iiia$, starting from the firm fundamental position of the tonic chord, does not produce a similar unfavorable effect.

14. THE NEGATIVE ELEMENT OF THE LEADING NOTE:
 $LN = 2, 4$

As its name indicates, the leading note has a distinct tendency to move to the adjacent tonic note and, in a lesser degree, to the adjacent submediant which is the tonic of the related minor chord. Hence, if the leading note appears in the first chord but is not contained in the subsequent chord, and does not step to one of these adjacent notes, there is a definite feeling that the leading note has not been properly dealt with.

Moreover whenever the leading note leaps more than a third or is transferred from any part of the first chord to the bass of the second chord, the same feeling arises.

Again, in case the leading note is present in both chords, thus strongly suggesting a dominant sequence (note that iii , V and $viib$ are dominant), while the sequence is not dominant in point of fact, the leading note is felt to be particularly out of place.

Thus we assign to the negative element of the leading note, LN , an index 2, except in the last case when we assign to it an index 4.

15. THE NEGATIVE ELEMENT OF STATIONARY NOTES:
 $SN = 2, 4$

When an extreme part is held in a sequence of two chords the effect is monotonous. Even if the bass rises or falls an

312 Mathematical Theory of Aesthetics

octave the effect remains much the same. A similar displeasing effect is felt when two inner voices are held, or when one inner voice is held and the leading note is left by one voice and taken again by the adjacent voice.

Furthermore, if two of the voices are interchanged in position, as C and E going to E and C respectively, a similar unfavorable effect is produced.

In these cases we shall say that the negative element of stationary notes SN is present and give SN an index 2 or 4, according as two parts or more than two parts are involved.

However we do not allow more than two voices to be actually stationary, and exclude the case of stationary bass and soprano voices, since chordal sequences are unsatisfactory under these circumstances.

16. THE NEGATIVE ELEMENT OF DISSONANT LEAP: $DL = 2$

The dissonant leap of a diminished fifth or of a minor seventh to or from the dominant although allowable, is not in itself agreeable, since the voice moves more naturally by a consonant interval than by a dissonant one.

In the case of such a dissonant leap we introduce a corresponding negative element of order DL of index 2.

17. THE NEGATIVE ELEMENT OF SIMILAR MOTION: $SM = 2$

If all the parts move up or if all move down, there is an undesirable effect of similarity of function produced by the similar motion, to which corresponds the last negative element SM with index 2.

It is usually possible to arrange chordal sequences of a given type so that not all the parts move in the same direction.

18. THE NEGATIVE ELEMENT OF BASS LEAP: $BL = 4$

If the perfect fourth above the bass is a note of a chord, the chord is in its second inversion and if felt not to be in normal position, since in normal position the third and not the fourth above the bass is found in the chord. In consequence the bass voice moves readily to the bass of such a chord only by step or by leap from a fundamental position; and it leaves such a chord position readily only by step.

In consequence, if a chord of this kind is approached or left by leap, but not from a fundamental position, the negative element DL is taken to be present and is assigned the index 4.

This completes our list of negative elements of order: FR , Mt , LN , SN , DL , SM , BL . All of these deal with negative æsthetic factors whose importance is scarcely questionable.

19. RECAPITULATION OF DEFINITION OF M FOR CHORDAL SEQUENCES

The æsthetic measure of a regular sequence of two chords is defined as the sum of the æsthetic measures of the two constituent chords m_1 and m_2 and of the transition value t :

$$M = m_1 + m_2 + t.$$

The transition value t is the sum of the following elements:

$$\textit{The Element of Resolution: } R = 4, -4$$

The element R is 4 when a dissonant first chord resolves on a consonant second chord, in accordance with the rule of section 6. If this resolution is not effected, R is -4 .

314 Mathematical Theory of Aesthetics

The Cadential Element: Cl = 4, -2

The element *Cl* is 4 if the sequence is of one of the following types:

V, viib \longleftrightarrow I, via ($7 \longleftrightarrow 1$ in viib \longleftrightarrow via), iii \longleftrightarrow vi,
IV \longleftrightarrow I, ii \longleftrightarrow vi.

The element *Cl* is -2 in the following types of sequence:
7 \rightarrow 1 in extreme part, but without cadence as above,
IVa \rightarrow via.

Otherwise *Cl* is 0.

This element *Cl* is not counted if *R* is counted.

The Element of Dominant Sequence: D = 4

The sequence involves two of the following chords ii, iii, viib, V, V7, V9, V11, V13, with viib \rightarrow ii, iii, V and ii \rightarrow iiia excepted.

In the non-excepted cases *D* is taken as 4, at least if the following further requirements are satisfied: in the sequences

ii \rightarrow viib, V; iii \rightarrow viib, V,

the mediant in ii and the submediant in iii move by step; in iiia \rightarrow V the mediant must be in the highest part of iiia.

For all other cases *D* is taken to be 0.

The Elements SF and RF; SF = 4, RF = 4

These elements *SF* and *RF* are present respectively when there is a sequence of two first inversions in which the bass moves up or down one or two steps, and when a fundamental position is followed by a first inversion with the root falling one step.

The Element of Progression: P = 2, 0, -2

The element *P* is 2 if two notes of the first chord can be

“borrowed” as consonant notes for the second chord. The borrowing of a dominant note for use in viib is permitted. The element P is also 2 when no notes of the first chord can be so borrowed, provided that each note of the first chord goes to a nearest note of the second chord except in so far as the bass note or an adjacent note or an octave of this note has proceeded to such a note, in which case the first note is free to move a step in the opposite direction; moreover the tonic is allowed to move a third, and the motion of a freed octave in the opposite direction is unrestricted.

Otherwise the element P is 0 if one consonant note only can be so borrowed or if there is resolution or if the sequence is dominant; P is -2 in every other case.

The Negative Element: $FR = 4$

The sequence involves a first inversion followed by a chord in fundamental position, with the root falling one step.

The Negative Element of the Mediant Chord: $Mt = 2, 4$

The element Mt is 2 if iii is present but the sequence is not of the types: $iiia \rightarrow Ia$ or 6, 1, * \rightarrow 3, 5, * with bass leap from 6 or to 3, or both; also if iii is not present but the sequence is of the second type: Mt is 4 if iii is present and the sequence is of one of these types.

The Negative Element of the Leading Note: $LN = 2, 4$

If the leading note leaps and is not found in the second chord, or leaps more than a third, or is transferred to the bass, LN is 2. If the leading note is in both chords of a non-dominant sequence LN is 4.

316 Mathematical Theory of Aesthetics

The Negative Element of Stationary Notes: SN = 2

The element SN is 2 if an extreme part or two parts are held stationary (inclusive of a leading note taken and left by adjacent voices) or interchanged.

If three parts are involved in this way, SN is 4.

The Negative Element of Dissonant Leaps: DL = 2

One or more of the parts leap an allowed dissonant interval (section 4).

The Negative Element of Similar Motion: SM = 2

All of the parts move up or down, but not in the same chord.

The Negative Element of Bass Leap: BL = 4

The sequence contains a second inversion which is approached or left by leap but not from a fundamental position of the first chord.

20. COMPARISON WITH PROUT'S CLASSIFICATION

Fortunately the æsthetic classification of all possible types of chordal sequences has been effected empirically; for instance, in Prout's *Counterpoint; Strict and Free*¹ a table of sequences involving the fundamental positions and first inversions of the triadic chords, is given. These are rated by Prout as "good," "possible," and "bad."

It is essential of course that the result of our theory be in fair agreement with these tabulated results which are the results of observation and experience. Indeed I have used this table as a basic aid in the æsthetic analysis of chordal sequences.

¹ Third edition.

In order to effect the comparison of Prout's table and our own results, the best positions of all the possible 144 types of chordal sequences involving the fundamental positions and first inversions of the triadic chords have been determined on the basis of the definition of æsthetic measure given above. These results are entered in the table at the end of this section as follows: at the head of each column appears the æsthetic measure; the chordal sequence in question is entered by means of the figures 1, . . . 7, for the respective roots, these being written in Roman or italic form according as a fundamental position or first inversion is indicated. Thus 47 would symbolize IVa \rightarrow viib.

It will be seen that there is complete agreement as far as could reasonably be expected, with M less than 6 and greater than 9 corresponding generally to the "bad" and "good" sequences respectively.

As Prout remarks in this connection (p. 30) "the words 'good,' 'possible' and 'bad' must not be taken as more than mere approximations."

In explanation of the æsthetic measures of the list, it should be remarked that in two cases (36 and 67) a chord of measure only 1 was used in order to secure harmonic progression; strictly speaking, only chords of measure at least 2 are to be employed. Furthermore in the sequences 37, 57, the second chord is regarded as the second inversion of V7 rather than as viib, since the dominant is borrowed from the preceding chord, and thus the negative element $BL = 4$ is attributed to these two sequences of the list.

21. GENERAL REMARKS

The above theory provides a method of theoretic comparison of all possible regular chordal sequences, such as

318 Mathematical Theory of Aesthetics

would be entirely impossible by means of any set of empirical negative rules, like those found in Prout (*loc. cit.*).

The substantial agreement of the theoretical ratings with Prout's table, together with the fact that the theory is based on familiar and verifiable elements, weighted in a simple way seems to me to furnish conclusive evidence in support of the general correctness of the theory. In fact no unfounded theory of this kind could hope to rate correctly 144 types of chordal sequences without important exception, any more than a simple cryptogram could apply to an arbitrary collection of 144 letters.

Undoubtedly the above theory could be considerably improved. However, I doubt whether any modification which takes account of more recent musical forms can be made as yet, since these forms have not become sufficiently well established. Indeed it is entirely conceivable that classical harmony will remain a permanent convenient norm from which interesting stylistic sallies are made in many directions, but to which there will be a tendency to return.

ÆSTHETIC MEASURE *M*
AND PROUT'S RATING OF CHORDAL SEQUENCES

Prout's Rating	Bad					Possible*					Good									
M =	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	32	73	54	43	13	23	21 ^b	13*	13*	12*	12	12	14	15	14	15	14		17	
	73				23	21	31 ^b	16*	12*	13*	16	16	15	25	14	65	17			
					31	34	36*	24*	16*	27*	24	21	32	27	15		51			
					34	32	46*	25*	23*	26*	26	25	41	41	51					
					35	37	61*	31*	24*	36*	26	21	45	53	54					
					43	52	64*	34*	26*	32*	23	24	41		56					
						63	61*	35*	36*	34*	37	25	45		57					
						65	63 ^b	42*	35*	31*	42	42	52		51					
							72*	53*	36*	46*	47	46	53		65					
							74*	63*	35*	43*	43*	52	54		71					
							75*		45*	56*	45	56	57							
							75*		46*	62*	41*	56	71							
									42*	61*	51	62								
									47*	64*	62	64								
									54*	72*	64	65								
									52*		67									
									53*		76									
									63*											
									61*											
									67*											
									62*											
									76*											
									74*											

The figures indicate the roots (1 = tonic, etc.): Roman for fundamental position, italic for first inversion. Thus 14 stands for Ia → IVb. For *M* between 5 and 10, superscripts b and g indicate "bad" and "good" respectively, according to Prout.